

Str. 55, Reši enačbe

34. a) $0,008 = \sqrt[3]{5}$

c) $\sqrt{5^{3-x}} \cdot \sqrt[3]{25^{x-1}} = 0,2$

35. a) $a^{2x} \cdot \sqrt{a^{x-13}} = \sqrt[3]{a^x}$

c) $x^{-1}\sqrt{4^{x+1}} : x^{+1}\sqrt{4^{x-1}} = \sqrt[3]{4^8}$

36. a) $2^{x+3}\sqrt{a^{3x-2}} - 2^{x+4}\sqrt{a^{3x-5}} = 0$

37. a) $4^{x+1} + 4^x = 320$

40. a) $7^{x+2} + 2 \cdot 7^{x-1} = 345$

c) $2 \cdot 3^{x-3} + 3^{x-2} + 3^{x-1} = 14 \cdot 3^5$

b) $x^{-2}\sqrt{3^x} = x^{-3}\sqrt{3^{1+x}}$

č) $1000 \cdot 10^x = \sqrt[3]{100^2}$

b) $9^x : \sqrt[3]{3^{x-1}} = 3^{x-1} \cdot \sqrt{3^{3x-4}}$

b) $3^{x+4}\sqrt{a^{x-3}} = 7^{x+8}\sqrt{a^{5x-6}}$

b) $2 \cdot 3^{x+1} - 4 \cdot 3^{x-2} = 450$

b) $3 \cdot 2^{2x-1} - 9 \cdot 2^x + 5 \cdot 2^{x-2} + 400 = 0$

Opomba: Priporočam, da najprej rešiš naloge 16, 17, 22, 23 na strani 53 in 54.

Enačbe eksponentne

Pomni:

Rešiti moramo eksponentne enačbe to so enačbe z neznanko v eksponentu.

V osnovi poznamo tri tipe eksponentnih enačb:

(1) $a^{f(x)} = a^{g(x)} \Leftrightarrow f(x) = g(x)$ (pri enakih osnovah izenačimo eksponent)

(2) $a^{f(x)} = b^{f(x)} \Leftrightarrow f(x) = 0$ (Pri enakih eksponentih izenačim eksponent z 0)

(3) $a^{f(x)} = b \Leftrightarrow \log a^{f(x)} = \log b$ (osnovi sta različni in eksponenta sta različna, tedaj enačbo logaritmiramo pri najbolj ugodni osnovi)

Rešitve:

34. a) $0,008 = \sqrt[3]{5}$

$$\frac{8}{1000} = 5^{\frac{1}{x}}$$

$$\frac{1}{125} = 5^{\frac{1}{x}}$$

$$5^{-3} = 5^{\frac{1}{x}}$$

$$\frac{1}{x} = -3 / :x$$

$$1 = -3x$$

$$x = -\frac{1}{3}$$

Uporabim zvezo:

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}$$

b) $x^{-2}\sqrt{3^x} = x^{-3}\sqrt{3^{1+x}}$

$$3^{\frac{x}{x-2}} = 3^{\frac{1+x}{x-3}}$$

$$\frac{x}{x-2} = \frac{1+x}{x-3}$$

$$x(x-3) = (x-2)(1+x)$$

$$x^2 - 3x = x + x^2 - 2 - 2x$$

$$-3x = -x - 2$$

$$-2x = -2$$

$$\underline{x = 1}$$

<p>c) $\sqrt{5^{3-x}} \cdot \sqrt[3]{25^{x-1}} = 0,2$</p> $5^{\frac{3-x}{2}} \cdot 5^{\frac{2(x-1)}{3}} = \frac{2}{10}$ $5^{\frac{3-x}{2} + \frac{2-x}{3}} = \frac{1}{5}$ $5^{\frac{3(3-x)+2(2x-2)}{6}} = 5^{-1}$ $\frac{3(3-x)+2(2x-2)}{6} = -1/6$ $9-3x+4x-4 = -6$ $x+5 = -6$ $\underline{x = -11}$	<p>č) $1000 \cdot 10^x = \sqrt[4]{100^2}$</p> $10^3 \cdot 10^x = 10^{\frac{4}{x}}$ $10^{x+3} = 10^{\frac{4}{x}}$ $x+3 = \frac{4}{x} / \cdot x$ $x^2 + 3x = 4$ $x^2 + 3x - 4 = 0$ $(x-1)(x+4) = 0$ $\underline{x_1 = 1} \quad \underline{x_2 = -4}$
<p>35. a) $a^{2x} \cdot \sqrt{a^{x-13}} = \sqrt[3]{a^x}$</p> $a^{2x} \cdot a^{\frac{x-13}{2}} = a^{\frac{x}{3}}$ $a^{2x + \frac{x-13}{2}} = a^{\frac{x}{3}}$ $2x + \frac{x-13}{2} = \frac{x}{3} / \cdot 6$ $12x + 3(x-13) = 2x$ $12x + 3x - 39 = 2x$ $15x - 2x = 39$ $13x = 39$ $\underline{x = 3}$	<p>b) $9^x : \sqrt[3]{3^{x-1}} = 3^{x-1} \cdot \sqrt{3^{3x-4}}$</p> $3^{2x} : 3^{\frac{x-1}{3}} = 3^{x-1} \cdot 3^{\frac{3x-4}{2}}$ $3^{2x - \frac{x-1}{3}} = 3^{x-1 + \frac{3x-4}{2}}$ $2x - \frac{x-1}{3} = x-1 + \frac{3x-4}{2} / \cdot 6$ $12x - 2(x-1) = 6x - 6 + 3(3x-4)$ $12x - 2x + 2 = 6x - 6 + 9x - 12$ $10x + 2 = 15x - 18$ $-5x = -20$ $\underline{x = 4}$
<p>c) $\sqrt{x-1} \sqrt{4^{x+1}} : \sqrt[3]{4^{x-1}} = \sqrt[3]{4^8}$</p> $4^{\frac{x+1}{2}} : 4^{\frac{x-1}{3}} = 4^{\frac{8}{3}}$ $4^{\frac{x+1}{2} - \frac{x-1}{3}} = 4^{\frac{8}{3}}$ $\frac{x+1}{2} - \frac{x-1}{3} = \frac{8}{3} / \cdot 3(x-1)(x+1)$ $3(x+1)^2 - 3(x-1)^2 = 8(x+1)(x-1)$ $3(x^2 + 2x + 1) - 3(x^2 - 2x + 1) = 8(x^2 - 1)$ $3x^2 + 6x + 3 - 3x^2 + 6x - 3 = 8x^2 - 8$ $12x = 8x^2 - 8$ $8x^2 - 12x - 8 = 0 / : 4$ $2x^2 - 3x - 2 = 0$ $D = b^2 - 4ac = 9 + 16 = 25$ $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{3 \pm 5}{4}$ $\underline{x_1 = 2}, \quad \underline{x_2 = -\frac{1}{2}}$	

$$36. a) \sqrt[2x+3]{a^{3x-2}} - \sqrt[2x+4]{a^{3x-5}} = 0$$

$$a^{\frac{3x-2}{2x+3}} - a^{\frac{3x-5}{2x+4}} = 0$$

$$a^{\frac{3x-2}{2x+3}} = a^{\frac{3x-5}{2x+4}}$$

$$\frac{3x-2}{2x+3} = \frac{3x-5}{2x+4}$$

$$(3x-2)(2x+4) = (2x+3)(3x-5)$$

$$6x^2 + 12x - 4x - 8 = 6x^2 - 10x + 9x - 15$$

$$8x - 8 = -x - 15$$

$$9x = -7$$

$$\underline{x = -\frac{7}{9}}$$

$$b) \sqrt[3x+4]{a^{x-3}} = \sqrt[7x+8]{a^{5x-6}}$$

$$a^{\frac{x-3}{3x+4}} = a^{\frac{5x-6}{7x+8}}$$

$$\frac{x-3}{3x+4} = \frac{5x-6}{7x+8}$$

$$(x-3)(7x+8) = (3x+4)(5x-6)$$

$$7x^2 + 8x - 21x - 24 = 15x^2 - 18x + 20x - 24$$

$$7x^2 - 13x - 24 = 15x^2 + 2x - 24$$

$$8x^2 + 15x = 0$$

$$x(8x+15) = 0$$

$$\underline{x_1 = 0} \quad 8x+15 = 0$$

$$\underline{x_2 = -\frac{15}{8}}$$

$$37. a) 4^{x+1} + 4^x = 320$$

$$4^x(4^1 + 4^0) = 320$$

$$4^x(4+1) = 320$$

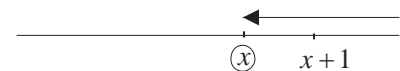
$$4^x \cdot 5 = 320 / :5$$

$$4^x = 64$$

$$4^x = 4^3$$

$$\underline{x = 3}$$

Tu si x in $x+1$ zamislimo na številski osi



in izpostavim skupno osnovo na najmanjši eksponent (najbolj levo na osi). Tu je to x .

$$b) 2 \cdot 3^{x+1} - 4 \cdot 3^{x-2} = 450$$

$$3^{x-2}(2 \cdot 3^3 - 4 \cdot 3^0) = 450$$

$$3^{x-2}(54 - 4) = 450$$

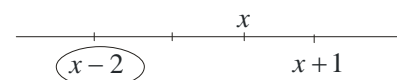
$$3^{x-2} \cdot 50 = 450$$

$$3^{x-2} = 9$$

$$3^{x-2} = 3^2$$

$$x-2 = 2$$

$$\underline{x = 4}$$



40. a) $7^{x+2} + 2 \cdot 7^{x-1} = 345$

$$7^{x-1}(7^3 + 2 \cdot 7^0) = 235$$

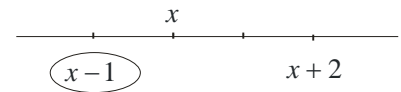
$$7^{x-1}(343 + 2) = 345$$

$$7^{x-1} = 1$$

$$7^{x-1} = 7^0$$

$$x - 1 = 0$$

$$\underline{x = 1}$$



b) $3 \cdot 2^{2x-1} - 9 \cdot 2^x + 5 \cdot 2^{x-2} + 400 = 0$

$$3 \cdot 2^{x-1} - 9 \cdot 2^x + 5 \cdot 2^{x-2} = -400$$

$$2^{x-2}(3 \cdot 2 - 9 \cdot 2^2 + 5 \cdot 2^0) = -400$$

$$2^{x-2}(6 - 36 + 5) = -400$$

$$-25 \cdot 2^{x-2} = -400$$

$$2^{x-2} = 16$$

$$2^{x-2} = 2^4$$

$$x - 2 = 4$$

$$\underline{x = 6}$$

Člen brez faktorja 2^{x+n} dam na desno!
Potem lahko na levi izpostavim 2 na najmanjši skupni eksponent.

c) $2 \cdot 3^{x-3} + 3^{x-2} + 3^{x-1} = 14 \cdot 3^5$

$$3^{x-3}(2 + 3 + 9) = 14 \cdot 3^5$$

$$3^{x-3}(2 + 3 + 9) = 14 \cdot 3^5$$

$$3^{x-3} \cdot 14 = 14 \cdot 3^5$$

$$3^{x-3} = 3^5$$

$$x - 3 = 5$$

$$\underline{x = 8}$$